REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE

3. REPORT TYPE AND DATES COVERED
FINAL 01 Apr 88 to 31 Dec 90

4. TITLE AND SUBTITLE
MODELING OF FREE VISOELASTIC JETS AND INSTABILITY MECHANISMS

5. FUNDING NUMBERS
AFOSR-88-0164
61102F 2304/A4

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8. PERFORMING ORGANIZATION REPORT NUMBER
AFOSR-TR- 91 0684

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
Bldg 410
Bolling AFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-88-0164

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The prinipal investigators, Greg Forest and Stephen Bechtel, have made progress on analytical, numerical, and experimental fronts in our studies of non-Newtonian fluids. The resources afford by this grant have contributed significanlty to these research efforts. In addition to summer support for the p.i.'s the AFOSR has partially supported Basab Maulik, a postdoc from Princeton University: Karen Bolinger, a 1989 Ph.D graduate under the advisement of Forest; Qi Wang, a 1991 Ph.D. graduate of forest's; and Gene Cao, a current Ph.D. candidate of Forest's who will gaduate in 1992.
We summarize the main resuslts which have been attained during this granting period an the publications which have resulted that acknowledge this AFOSR grant. The results will be separated into an analytical category, a numerical category, and an experimental category.

14. SUBJECT TERMS

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

Final Report

AFOSR Grant 88-0164

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We summarize the main results which have been attained during this granting period and the publications which have resulted that acknowledge this AFOSR grant. The results will be separated into an analytical category, a numerical category, and an experimental category.

I. Analytical Studies

We now have produced two asymptotic theories (pointwise and averaged) for slender free surface non-Newtonian jets. This analysis: (a) reduces spatial dimensions from three to one; (b) yields a seven parameter family of closed systems of pde's in one space, one time that govern slender free non-Newtonian jets; (c) establishes formal consistency to all orders of perturbation for these 1-D models; (d) produces explicit models of a catastrophic change-of-type from hyperbolic to mixed hyperbolic/elliptic type. These results are contained in references [1-6].

These studies have spawned several new ongoing projects related to: numerical loss-of-evolution, change-of-type in the full 3-D Maxwell models, physical validity and consequences of a mathematical change-of-type, numerical detection and consequences of change-of-type, physically meaningful regularizations that produce well-posedness in

an ill-posed regime, the analysis of convergent and stable numerical codes, and molecular dynamics simulations of dilute polymer solutions. These current projects involve collaborations with M. Gunzburger, VPI, James Turner, Hampton University, and C. Jayaprakash, Department of Physics, Ohio State University, as well as two Ph.D. students of Forest, Q. Wang and J. Cao.

II. Numerical Studies

We have several interconnected numerical studies, involving the p.i.'s, Maulik, Wang, Cao, and more recently Gunzburger and Turner.

Wang has focussed on the numerical simulation of specific asymptotic model systems. These codes compute well-posed boundary value problems for steady solutions based on the characteristics of the pde systems. Steady states are computed, linearized stability of steady states and effects of boundary conditions are computed, and the full time-dependent codes are developed and run in a neighborhood of steady states to explore basins of stability and robustness with respect to parameters. The systems of pde's have change-of-type behavior, and there are nontrivial coding issues related to this. It is here that mathematically deduced physical regularization of a change-of-type is critical. We know that in our 1-D model problems the retardation effects regularize the elliptic change-of-type from high Weissenberg numbers, but only in a neighborhood of constant solutions. The development of a stable code for the regularized equations is in progress. This project is prototypical of the 3-D loss-of-evolution pathologies.

Cao has developed numerical codes which allow for the investigation of the asymptotic validity of leading order slender jet solutions. Many solutions to "thin filament" models have been presented in the engineering and polymer processing literature since 1969. Our work was the first to demonstrate that these models can be obtained as the leading order problems in a *formally* valid slender jet expansion (see § I); now in the paper with Cao [8] we are the first to investigate, on a solution-by-solution

basis, which solutions are indeed asymptotically valid. Briefly, we find that, although all of the models are formally valid, particular solutions to these models may be valid (corrections to the leading order behavior remain small) or invalid (corrections become sufficiently large to disrupt the asymptotic ordering on which the theory is based).

Maulik has numerically studied laminar transition in 2-D and 3-D Maxwell fluids. These studies involve massive codes and the results are now beginning to get published. We anticipate comparison of these macroscopic simulations with molecular dynamics simulations of dilute polymer solutions by C. Jayaprakash, Department of Physics, O.S.U. The goal of this project is to model realistic dynamics of the large polymer molecules and ultimately infer corrections to Maxwell fluid models which are connected to realistic polymer dynamics.

III. Experimental Studies

Experiments have been performed involving free surface jets of four very different types of fluids: (1) water, (2) viscoelastic aqueous biopolymer solutions in three different concentrations (Xanthan gum), (3) a viscoelastic aqueous surfactant solution (Ethoquad/sodium salicylate), and (4) water/detergent solutions.

The experiments imply that:

1) robust, slender jet profiles exist, so that the assumptions of our asymptotics are supported;

2) noticeable effects due to material (viscoelastic) properties are documented in otherwise identical experiments.

An open problem in rheology: How does one measure relaxation time Λ_1 in elongational flows? (Measurement of elongational response in fluids has proven difficult because regions of elongational flow are in general difficult to isolate and involve free surfaces, which complicates the analysis. Elongational response cannot be inferred from the more easily measured shear response of a fluid.) Our asymptotic models for free jet

flow (which is elongational) *predict* experimentally measurable effects as functions of Λ_1 for other parameters fixed. This poses an intriguing *inverse problem* from our analysis: we infer Λ_1 by comparing experimental properties of the free jet flow (e.g., the wavelength and amplitude of the observed nonlinear spatial oscillation in the steady profile of a jet issuing from an elliptical orifice, and the growth or decay of these quantities in successive cycles down the jet) with the corresponding value of Λ_1 that produces the same properties in the 1-D model.

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PRESENTATIONS

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"1-D Closure Models for Slender 3-D Viscoelastic Free Jets with Elliptical Cross Section," AIAA/ASME/SIAM/APS 1st National Fluid Dynamics Congress, Cincinnati, Ohio, July 1988; paper in the proceedings on pp. 2062-2069.

"1-D Closure Models for Newtonian and Viscoelastic Free Jets: Lowest Order Equations and Higher Order Corrections," 41st Annual Meeting of the American Physical Society, Division of Fluid Dynamics, Buffalo, New York, November 1988; abstract on page 2245 of the program.

"The Effects of Viscoelastic Properties on the Oscillation of Elliptical Free Jets," 60th Annual Meeting of the Society of Rheology, Gainesville, Florida, February 1989; abstract on page 40 of the program.

"Asymptotic Closure Models for Non-Newtonian Fluid Jets," invited talk at the Applied Mathematics Colloquium, University of Colorado, Boulder, September 1989.

"Experimental and Theoretical Studies of the Behavior of Viscoelastic Free Jets from Elliptical Orifices," 61st Annual Meeting of the Society of Rheology, Montreal, Canada, October 1989.

"The Oscillation of Elliptical Inviscid and Newtonian Jets: Effects of Surface Tension, Inertia, Viscosity, and Gravity," Winter Annual Meeting of the ASME, San Francisco, California, December 1989.

"Nonlinear Oscillations in Newtonian and Viscoelastic Free Surface Jets," invited talks at the Ohio State University Engineering Mechanics Seminar, Columbus, February 1990, and the University of Michigan Mechanical Sciences Division Seminar, Ann Arbor, March 1990.

"Observations and Analysis of Newtonian and Non-Newtonian Free Surface Jets," invited talk at the University of Rochester Mechanical Engineering Colloquium, Rochester, New York, March 1990.

"Newtonian and Viscoelastic Free Jet Behavior," presented at the Eleventh U.S. National Congress of Applied Mechanics, Tucson, Arizona, May 21-25, 1990.

"On the Validity of Viscoelastic Slender Jet Models," presented at the 62nd Annual Meeting of the Society of Rheology, Santa Fe, New Mexico, October 21-25, 1990.

"The Break-up of Viscoplastic Filaments," and "Stability of Fiber-spinning Processes," presented at the Symposium on Interfacial Phenomena in Viscoelastic Flows, 1991 ASME Applied Mechanics and Biomechanics Summer Conference, The Ohio State University, June 16-19, 1991.